

Preface

Ecosystems are marvelous assemblages of individuals that grow, reproduce, interact with one another, move about in space, and eventually die. Ecosystems also provide essential services to humans, from oxygen production and recycling of organic matter to food provision and pollination of agricultural crops. At the same time, ecosystems are in peril from human activity, such as overexploitation, landscape fragmentation, and various effects of global change. Spatial ecology aims to understand the role that individual movement, population interaction, and landscape characteristics play in generating the patterns of species distributions that we observe in space and time, in particular questions of population persistence, population spread, and stability.

Dynamic mathematical models are powerful tools for understanding natural phenomena in the physical sciences and increasingly also in the life sciences. Mathematical methods, from the rigor of model formulation to the depth of analysis and the power of computation, are indispensable when dealing with the wide range of spatial and temporal scales that are inherent in many of the most fundamental questions of spatial ecology. Dynamical systems are also fascinating objects to study in their own right. The interaction between dynamical systems models and their applications in ecology and other fields is a constant source of mutual challenges and inspiration. The vision of this book is to introduce the reader to a class of models known as *integrodifference equations* and show how the fascinating, interdisciplinary circle of observation, modeling, analysis, and interpretation enhances ecological understanding and mathematical theory at the same time.

Integrodifference equations are models for the temporal evolution of the density of one or more populations from one generation to the next. These models are tailor-made to adequately represent the dynamics of organisms with a particular life cycle, where population growth and individual dispersal occur separated in time but synchronized in the population. Ecological examples contain many plant and insect species, particularly in temperate climates. Mathematically speaking, integrodifference equations define discrete dynamical systems (recursions) on some appropriate function space. Their mathematical analysis and ecological application have seen great progress in the past three decades. This book provides the first comprehensive

introduction to the subject and serves as a reference guide to the, by now, sizable literature on all aspects of these equations.

The book is divided into three parts:

I Basics and Foundations. Chaps. 1–8 contain the most important aspects of modeling with and analysis of integrodifference equations, using the simplest possible scenarios. The focus is on extinction, persistence, stability, and spread of a single species in simple landscapes. Basic ideas for numerical approaches are provided.

II Applications and Approximations. Chaps. 9–12 consider various aspects of real-world applications. The simplifying assumptions from the first part are modified where necessary to more adequately describe realistic situations. Various methods of model simplification are presented.

III Extensions and Challenges. Chaps. 13–17 discuss substantial and significant extensions to the simpler models in order to tackle more challenging biological questions, such as the dynamics of stage-structured populations, interacting populations, and spatial and temporal variation in the environment.

Throughout this book, questions of population dynamics and their application to real-world systems motivate all models and mathematical analysis, and all theoretical and computational results are discussed in relation to ecological theory and implications. The greatest progress is made where ecology and mathematics come together to inspire each other toward deeper understanding in each discipline and their interplay.

Acknowledgments

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Mark Kot, together with various collaborators, wrote the foundational articles that have inspired so much of the research on integrodifference equations, including my own. He has since produced many more beautiful and insightful articles on the subject. I admire the clarity, precision, and elegance in his works. He and Mike Neubert have graciously shared with me many insights and suggestions on integrodifference equations over the years. I also thank all of the participants of the

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Several students have contributed to this book in various ways. Jeff Musgrave meticulously collected the relevant literature until 2014; Dominic Brass provided an updated comprehensive list in 2017. The students of my topics courses on integrodifference equations in 2015 and 2018 provided feedback for drafts of some chapters, in particular Adèle Bourgeois, Jason Bramburger, and Alessandro Selvitella. I am also grateful to the Technische Universität München for awarding me a John von Neumann visiting professorship in 2017. Several chapters of the book were written and revised during my time there and with the help of the students who attended my classes. Mark Kot and his students Nora Gilbertson, Benjamin Liu, and Kelsey Marcinko read the penultimate version of the book, gave many helpful suggestions, and helped me fix some inconsistencies.

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A book on any research subject, once in print, is fixed and cannot evolve with the progress of the field that it aims to describe. Christina Cobbold and I, with the help of Wenyan Li, have created an online resource that includes a searchable reference list for integrodifference equations. We aim to maintain this website as an evolving repository for scientific publications, video presentations, computational tools, and teaching resources on integrodifference equations. It is accessible at

<https://integrodifference.frithjof.ca/>

We thank the University of Ottawa for hosting this site. We welcome all constructive comments and will post additions and corrections to the book at this site.

My greatest hope is that this book and the accompanying online resource will inspire new research directions in the field of integrodifference equations and create a community of mathematicians, modelers, and ecologists who transcend their disciplinary boundaries to advance knowledge in all.

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