

Recall

$$N_{t+1}(x) = \int_{-\infty}^{\infty} K(x-y) F(N_t(y), y) dy$$

no heterogeneity

Landscape heterogeneity and the spread of forest insects

heterogeneity

Frithjof Lutscher
University of Ottawa

University of Ottawa

March 2017

Forest Insects can do massive damage Aukema et al 2011

OPEN ACCESS Freely available online

PLoS one

Economic Impacts of Non-Native Forest Insects in the Continental United States

Juliann E. Aukema^{1*}, Brian Leung^{2,3}, Kent Kovacs⁴, Corey Chivers², Kerry O. Britton⁵, Jeffrey Englin⁶, Susan J. Frankel⁷, Robert G. Haight⁸, Thomas P. Holmes⁹, Andrew M. Liebhold¹⁰, Deborah G. McCullough¹¹, Betsy Von Holle¹²



Gypsy Moth



Emerald Ash borer



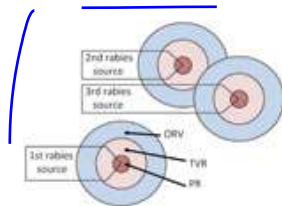
Asian Longhorn beetle



Beech scale

Ways to control?

- large-scale chemicals: kill the pest
expensive, harmful, limited success Liebhold and Tobin 2008
- biological control agent
- local detection and control Sharov et al 2002
 - barrier zones ('detect early and act locally')
 - manipulation of landscape structure
 - 'fire breaks' and 'ring vaccination' With 2002



Silvicultural measures Liebhold 2012

- reduce or remove resources
 - if organism is host specific
Brockerhoff et al 2010
 - Ontario ash-free zone failed
10 × 30 km
 - theoretical and empirical support
With 2004, Rigot et al 2014
 - buffer zones around wind throw
Nikolov et al 2014

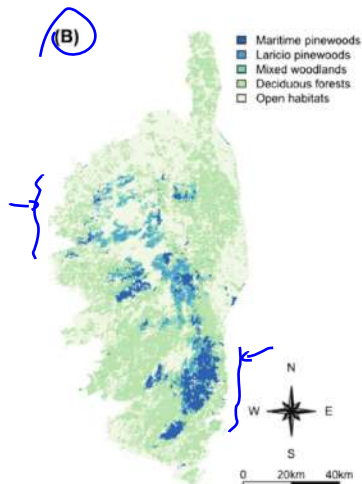
- thinning: increase tree vigor

Liebhold 2012

- increase diversity

Brockerhoff et al 2006, Jactel et al 2005, Rigot et al 2014

- resource concentration hypothesis
- enemy release hypothesis



Organisms adjust movement behaviour

Behaviours?

Edge effects

- Insects: increase residence time in host patch

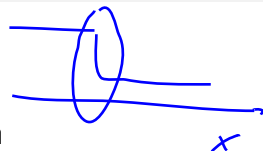
Ries and Debinski, Schultz and Crone, Reeve and Cronin ...

- Birds: gap crossing probability

Creegan and Osborne, Robertson and Redford...

- Mammals: carnivores and linear features

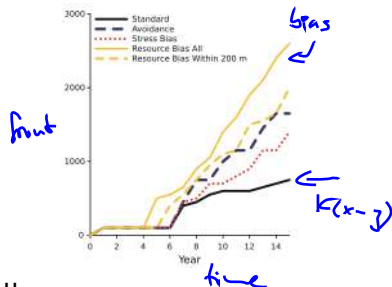
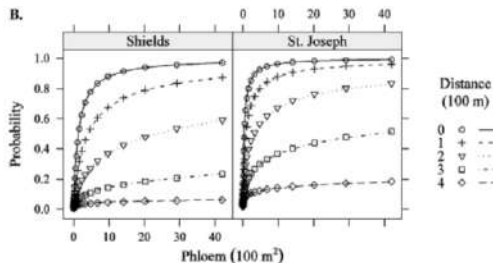
Whittington et al, McKenzie et al...



Organisms adjust movement behaviour

Invasion-dynamic consequences

- observation of increased dispersal distances Siegert et al 2010
- simulation shows faster spread at equal total phloem Mercader et al 2011 a,b



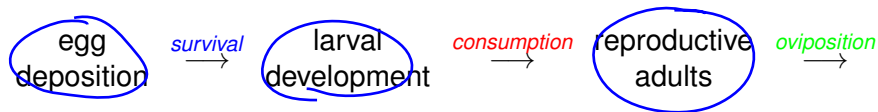
Make a model!

Overview

- 1 The model
 - Population dynamics
 - Movement
- 2 Analysis
 - Persistence
 - Spread
- 3 Results
 - Movement and persistence
 - Movement and spread rate
- 4 Discussion



Population dynamics



egg survival: s

phloem requirement: \bar{P}

egg production: r

phloem reduction: w

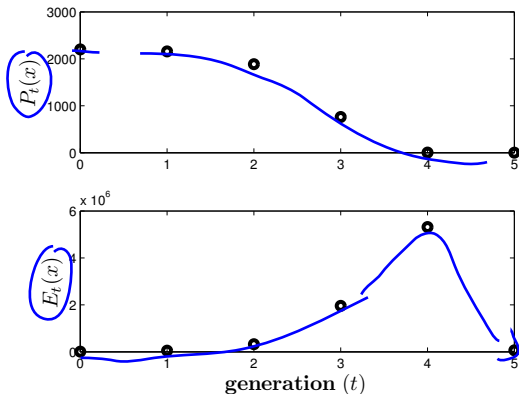
eggs
$$E_{t+1} = \left[\frac{r}{2} \left(1 - e^{-\frac{P_t}{\bar{P}}} \right) \right] s E_t,$$

phloem
$$P_{t+1} = P_t e^{-w s E_t}.$$

No re-growth of phloem

Dynamics of the population model

$$\begin{cases} E_{t+1} = \frac{r}{2} \left(1 - e^{-P_t/\bar{P}}\right) s E_t \\ P_{t+1} = P_t e^{-wsE_t} \end{cases}$$



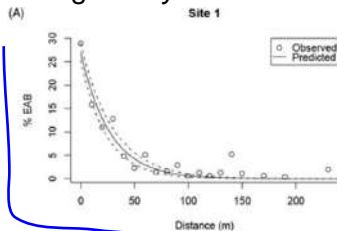
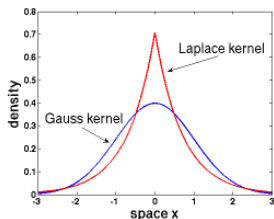
Musgrave 2013

The growth-dispersal model Musgrave and L (2017) Ecology

$$E_{t+1}(x) = \int K(x, y) \frac{r}{2} \left(1 - e^{-P_t(y)/\bar{P}}\right) s E_t(y) dy,$$

$$P_{t+1}(x) = P_t(x) e^{-wsE_t(x)}.$$

Dispersal kernel $K(x, y)$: probability of moving from y to x



Heterogeneity?

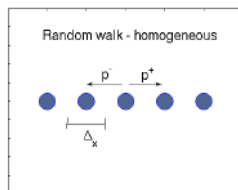
Mercader et al 2009

Movement behaviour – the random walk TURCHIN (1998)

p^\pm : right/left probability

ξ : space step

τ : time step



Master equation:

$$u(t + \tau, x) = [1 - (p^+ + p^-)]u(t, x) + p^+ u(t, x - \xi) + p^- u(t, x + \xi)$$

Diffusion limit ($p^+ = p^-$):

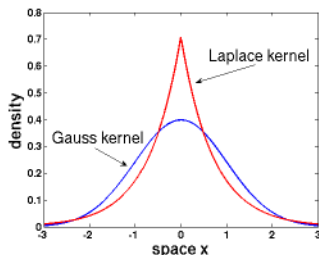
$$\frac{\partial}{\partial t} u(t, x) = D \frac{\partial^2}{\partial x^2} u(t, x) \quad D = \lim_{\tau, \xi \rightarrow 0} \frac{(p^+ + p^-) \xi^2}{2\tau}$$

Kernels in homogeneous landscape

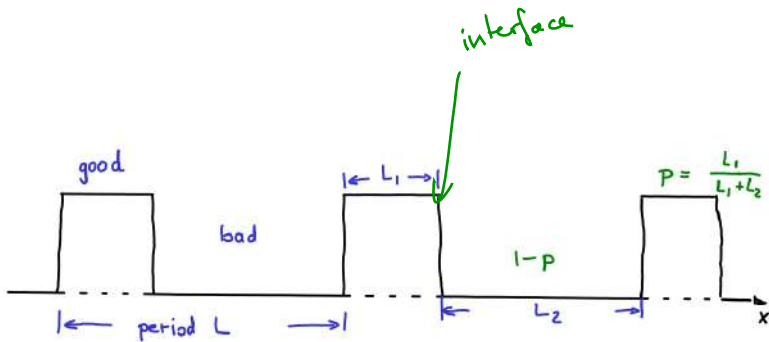
$$\frac{\partial}{\partial t} u(t, x) = \underbrace{D \frac{\partial^2}{\partial x^2} u(t, x)}_{\text{movement}} - \underbrace{m(t) u(t, x)}_{\text{oviposition}}, \quad u(0, x) = \delta(x)$$

$$K(x, 0) = \int_0^\infty m(t) u(t, x) dt$$

- 1 $m(t) = \delta(t - t_0)$
 \Rightarrow Gaussian
- 2 $m(t) = m$
 \Rightarrow Laplace



Two types of patches



Shigesada et al (1986) TPB

$$\frac{\partial}{\partial t} u = \frac{\partial^2}{\partial x^2} (D(x)u) - m(x)u$$

$$D = \begin{cases} D_1 & \text{if } x \in \text{good} \\ D_2 & \text{if } x \in \text{bad} \end{cases}$$

Equations and matching conditions

Good patches

$$\frac{\partial u}{\partial t} = D_1 \frac{\partial^2 u}{\partial x^2}$$

Bad patches

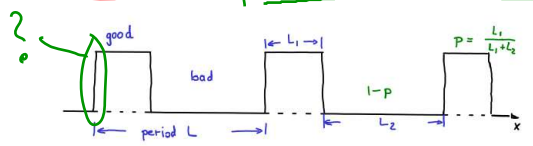
$$\frac{\partial u}{\partial t} = D_2 \frac{\partial^2 u}{\partial x^2}$$

Remember the smoothing property?

continuous flux

$$\lim_{x \rightarrow 0^-} u(t, x) = \lim_{x \rightarrow 0^+} u(t, x)$$

$$\lim_{x \rightarrow 0^-} D_1 \frac{\partial}{\partial x} u(t, x) = \lim_{x \rightarrow 0^+} D_2 \frac{\partial}{\partial x} u(t, x)$$



Heterogeneity II: Movement at interfaces

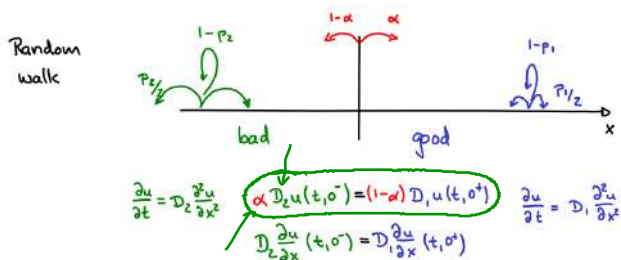


$$u(\Delta x, t + \Delta t) - \Delta x = \overset{1-p_2}{\cancel{v}} u(0x, t) \Delta x + \sum p_1 u(2\Delta x, t) \Delta x + u(0, t) \Delta x_0 \cdot \alpha$$

$$\rightarrow u(0, t + \Delta t) \Delta x_0 = \sum p_1 u(\Delta x, t) \Delta x + \sum p_2 u(-\Delta x, t) \Delta x$$

Ovaskainen and Cornell (2003), Maciel and Lutscher (2013)

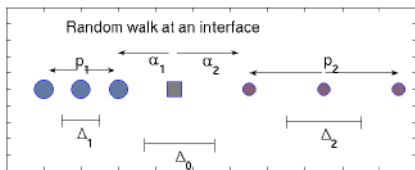
Heterogeneity II: Movement at interfaces



Remember? Matching condition $u(t, 0^-) = u(t, 0^+)$

Movement behaviour – the interfaces MACIEL AND L (2013) AM NAT

α_j : probability to move into patch (type) i



within-patch equations

$$\frac{\partial}{\partial t} u_1 = D_1 \frac{\partial^2}{\partial x^2} u_1$$

$$\frac{\partial}{\partial t} u_2 = D_2 \frac{\partial^2}{\partial x^2} u_2$$

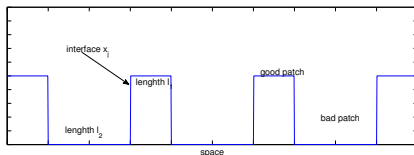
interface conditions

$$D_1 \frac{\partial}{\partial x} u_1(t, 0^-) = D_2 \frac{\partial}{\partial x} u_2(t, 0^+) \quad \checkmark$$

$$u_1(t, 0^-) = \frac{\alpha_1 D_2}{\alpha_2 D_1} u_2(t, 0^+) \quad \checkmark$$

Movement behaviour – dispersal kernel

Musgrave and L (2014a,b) JMB



Continuous movement, egg deposition, and mortality

$$\frac{\partial}{\partial t} u = D_j \frac{\partial^2}{\partial x^2} u - m_j u - \mu_j u, \quad x \neq x_j$$

Interface conditions $u(x_j^-) = k_j u(x_j^+)$

Dispersal kernel

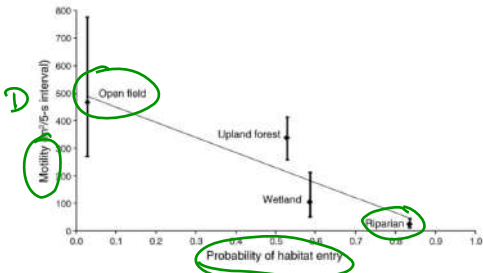
$$K(x, y) = \int_0^{\infty} m(x) u(t, x) dt$$

Ragged kernels Musgrave and L (2014a,b) JMB

bad: $D_2 > D_1$ good
 $\alpha > \frac{1}{2}$, $1 - \alpha < \frac{1}{2}$

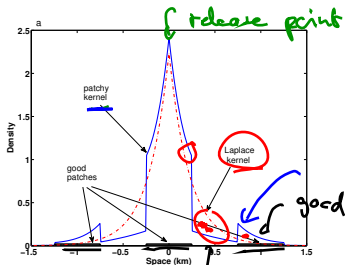
Butterfly

Satyroides appalachia



Kuefler et al 2010

α



$$\text{oviposits} = m(x) = \begin{cases} m_1 & \text{in good} \\ m_2 < m_1 & \text{in bad} \end{cases}$$

The growth-dispersal model Musgrave and L (2017) Ecology

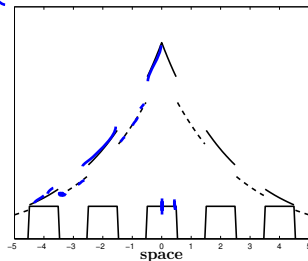
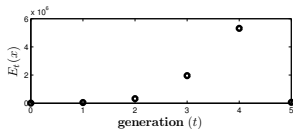
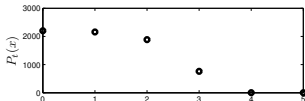
- E_t in # eggs/km
- P_t in m^2/km of phloem

pred-prey relationship

$$E_{t+1}(x) = \int K(x, y) \frac{r}{2} \left(1 - e^{-P_t(y)/\bar{P}}\right) s E_t(y) dy,$$

$$P_{t+1}(x) = P_t(x) e^{-wsE_t(x)}$$

no dispersal



Parameter estimation for EAB

Parameters for population dynamics

- $r = 70$ (eggs per female)
- $s = 0.55$ (probability of winter survival)
- $P_0 = 2200 \text{ m}^2/\text{km}$ (initial phloem density)
- $\bar{P} = 5000\text{m}^2/\text{km}$ (phloem requirement)
- $w = 0.000005\text{km}/\text{larvae}$ (impact on phloem)

Dispersal parameter in homogeneous landscape

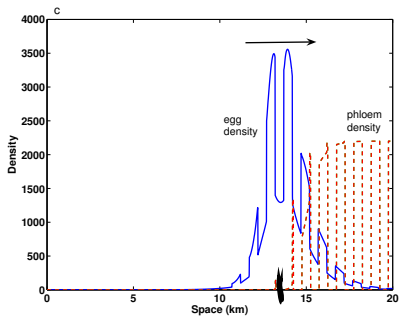
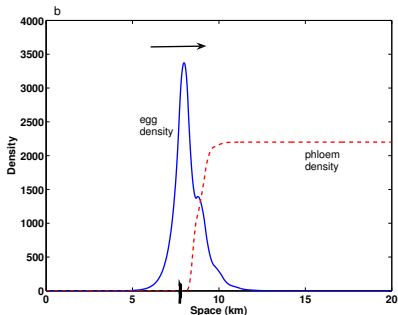
- mean dispersal distance of 0.22km
- $m = 2$ per day (ovipositioning rate)
- $D \approx 0.1\text{km}^2/\text{day}$

Anuniewicz et al 2008; Rutledge and Keena 2012; Crosthwaite et al 2011; McCullough and Siegert 2007; Mercader et al 2009, 2011, Siegert et al 2010

First numerical observation Musgrave and L (2017) Ecology

Initial condition: 70 eggs near $x = 0$

$$L_1 = L_2$$



$$c_h = 0.82 \text{ km/year}$$

Mercader et al (2016): $0.4-0.7$ (small) $1.2-1.7$ (large)

$$D_2 = 3D_1, m_2 = 0.5m_1, \alpha = 0.5$$

Analysis?

Challenges

- Numerics
- Discontinuous functions
- Non-cooperative system, equivalent to

$$E_{t+1}(x) = \int k(x, y) \frac{rs}{2} \left(1 - \exp \left(- \frac{P_0}{P} e^{-sw \sum_0^{t-1} E_k(y)} \right) \right) E_t(y) dy$$



Richard

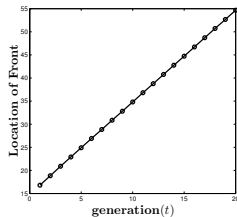
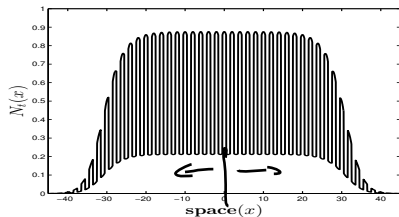
$$1 - e^{-\epsilon}$$

Opportunities

- Linearize
- Stability conditions
- Travelling periodic waves
- Homogenization

Travelling periodic waves

Pattern of spread



Musgrave and L (2014)

Travelling periodic waves Shigesada et al (1986)

The TPW ansatz for speed c and shape s

$$\underline{E_t(x) = e^{-s(x-ct)}\theta(x)}, \quad \theta(x) = \theta(x + L)$$

in the linear equation

$$\| E_{t+1}(x) = \int_{\mathbb{R}} K(x, y)r(y)E_t(y) dy$$

gives

$$\left[e^{sc}\theta(x) = \int_{\mathbb{R}} K(x, y)^{(x-y)} r(y) e^{s(x-y)}\theta(y) dy \right]$$

\implies Dispersion relation $c = c(s)$

$$K(x+L, y+L) = K(x, y)$$

$\implies \theta''$

Dispersion relation Musgrave and L (2014)

$$\frac{q_1^2 + q_2^2}{2q_1 q_2}$$

Relationship between speed c and steepness s of a TPW

$$\frac{q_1^2 + (q_2 \bar{z})^2}{2\bar{z}q_1 q_2} \sinh(q_1 l_1) \sinh(q_2 l_2) + \cosh(q_1 l_1) \cosh(q_2 l_2) = \cosh(sL)$$

with

$$q_i = \sqrt{\sqrt{D_i/m_i} [1 - \exp(-sc)r_i]}$$

and

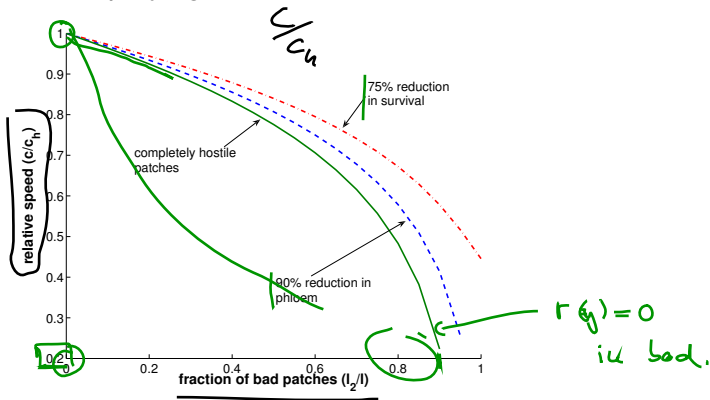
$$\bar{z} = (1 - \alpha)/(1 + \alpha), \quad l_1 + l_2 = L$$

\downarrow
 D_i
 m_i

Results: Landscape-independent movement

Scenario: inoculation, spraying

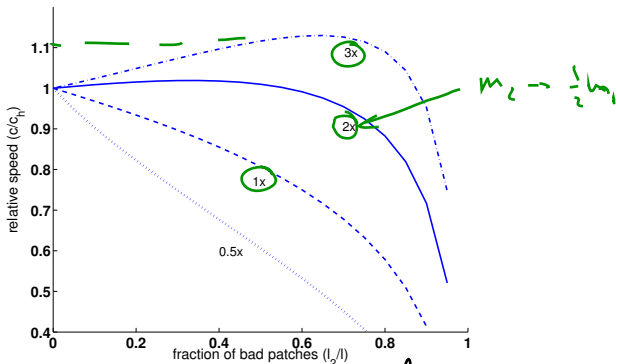
c_h : speed in homog. landscape



Laplace kernel

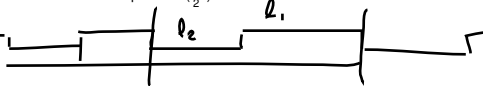
Results: Landscape-dependent diffusion

Scenario: detectable landscape alteration, no patch preference



$D_2 = n \times D_1$
bad patches
hostile

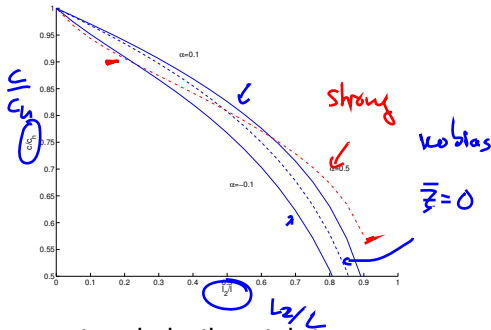
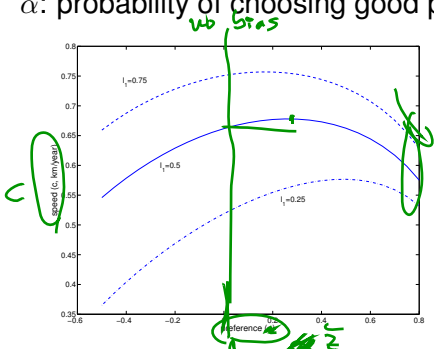
For ovipositioning: $D_i/m_i = \text{const.}$



Results: Patch preference

Scenario: Edge behaviour

α : probability of choosing good patch

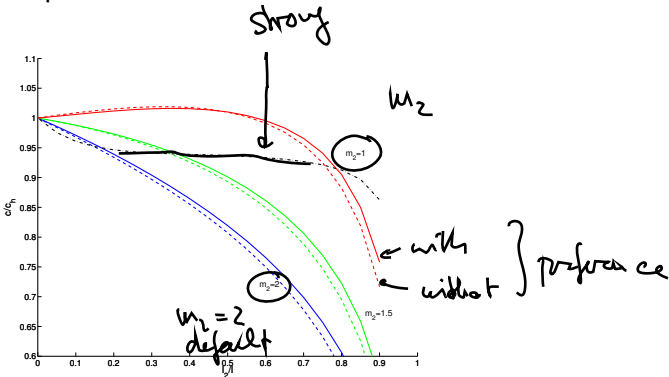


equal movement and oviposition parameters in both patch types
bad patches hostile

$$\alpha = \frac{1}{2} \leftrightarrow \bar{z} = 0$$

Results: Combined effects

oviposition and patch preference



- no preference (dashed)
- weak preference (solid)
- strong preference (dash-dot)

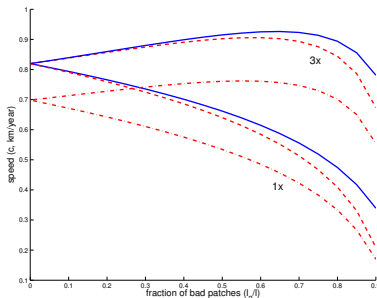
Results: Dispersal-related mortality

Scenario: hazard or energy requirements for dispersal

no mortality (solid)

low, equal mortality
(dash-dot)

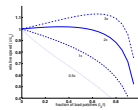
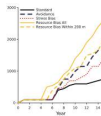
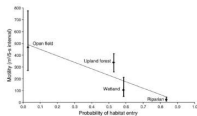
high bad-patch mortality
(dashed)



- persistent effect
- mortality in good patches counts

Summary and Discussion

- Invasives and control measures
 - Explore indirect measures Brockerhoff et al 2010
 - thinning, removal, diversity Muzika and Liebhold 2000, Jactel et al 2006, Liebhold 2012
 - fragmentation With 2002, 2004
- Individual movement response to control measures
 - Empirically observed Ruefler et al 2010
 - heuristically simulated Mercader et al 2011a,b
 - analytically investigated
- Exploitation versus exploration
 - higher diffusion, lower oviposition in bad patches



Summary and Discussion

- Edge behaviour
 - Emerald Ash borer
 - Asian longhorned beetle (strong flight, slow spread)
 - beech scale (wind dispersed, fast spread)
- Integrate optimal foraging with spatial spread models



wind

Summary and Discussion

- Analytical challenges
 - width and total mass of the pulse ↙
 - linear determinacy ↙
 - existence theory for discontinuous waves ↙
- Allee effect
- Stepping stones