Integrodifference equations in spatial ecology

Lecture 1: What, Why, How?

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Hallo!

Spatial Ecology

endangered species

Find examples!

Spread

- conservation
- invasion
- patterns
- inherently spatial
- inherently multi-scale
- mechanistic/process-based, phenomenological, correlation

Where are individuals?

Habitat suitability map

Integrodifference equation

- deterministic vs stochastic ?
- continuous vs discrete time
- continuous vs discrete landscape
- individuals vs density

Time cont

⇓

Reaction-diffusion eq.

Landscape discrete:

time cont.

set of ODE

time discrete

set of difference eq.

Modelling the life cycle

- discrete generations
- non-overlapping
- separated growth and dispersal
- synchronized

t: generation

generation n and $n+1$ don't see each other

within the population

separated in time

× space

Density $N_t(x)$.

$f(N_t(x))$.

annual plants.

$\boxed{N_{t+1}(x)}$

move from y to x.

Modelling the life cycle

Seeds

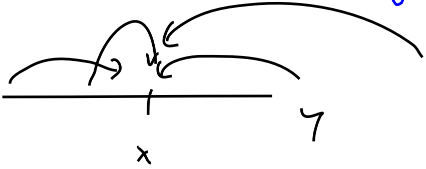
seeds produced

$N_t(y) \longrightarrow F(N_t(y)) \longrightarrow N_{t+1}(x)$

$k(x,y)$ is prob. of moving from y to x

$$\boxed{N_{t+1}(x)} = Q[N_t](x) = \int_\Omega K(x,y) F(N_t(y), y)\, dy$$

spatial heterogeneity

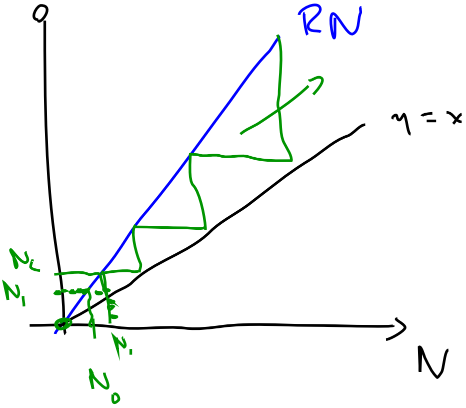


Discrete dynamical system

Growth functions, steady states, stability, cobwebbing

- linear $F(N) = RN$
- Beverton-Holt
- Ricker/logistic
- Allee

$F(N)$



$$N_{t+1} = F(N_t)$$

$$N_{t+1} = R^t N_0$$

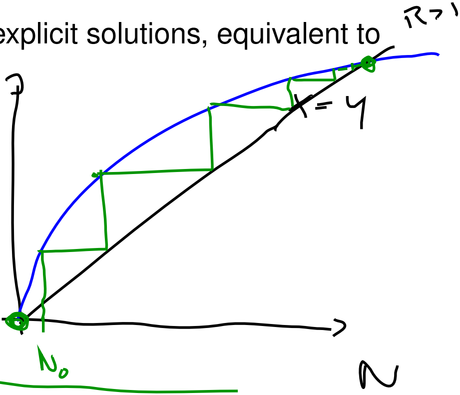
$F(0) = 0$

$\frac{N_{t+1}}{N_t}$ = per capita reproduction = R

Growth functions, steady states, stability, cobwebbing

- linear
- Beverton-Holt $F(N) = \frac{R}{1+\kappa N} N$ (explicit solutions, equivalent to Verhulst)
- Ricker/logistic
- Allee

f



steady states: $F(N)=N$

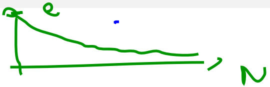
$N=0$ unstable

$N = \frac{R-1}{\kappa}$ is stable

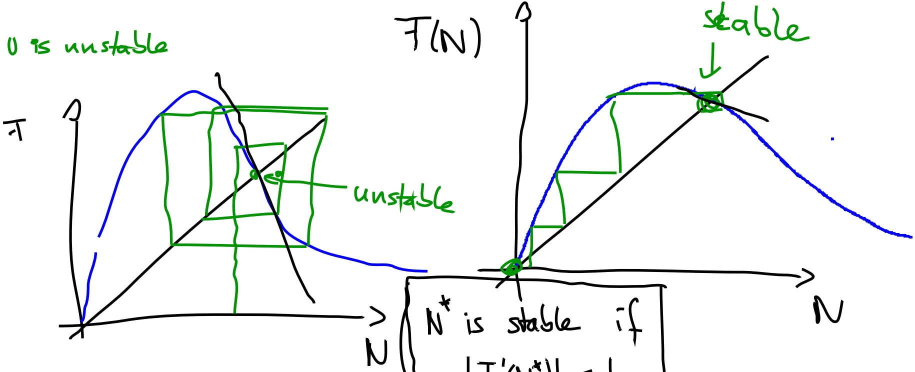
$$\frac{N_{t+1}}{N_t} = \frac{R}{1+\kappa N_t}$$

Growth functions, steady states, stability, cobwebbing

$$\frac{N_{t+1}}{N_t} = e^{r(1-N)}$$



- linear
- Beverton-Holt
- Ricker/logistic $F(N) = N\exp(r(1-N))$ or $F(N) = (r+1)N - rN^2$
- Allee

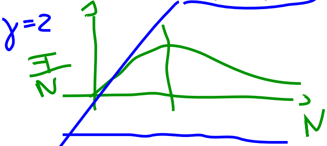


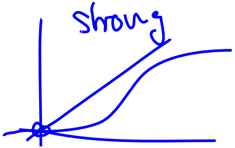
N^* is stable if $|F'(N^*)| < 1$

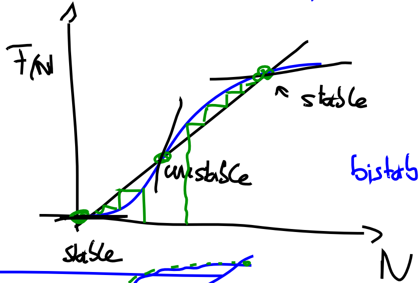
Growth functions, steady states, stability, cobwebbing

- linear
- Beverton-Holt
- Ricker/logistic
- Allee $F(N) = \frac{RN^\gamma}{1+(R-1)N^\gamma}$

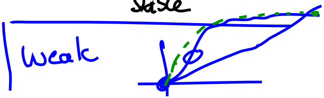
Per capita growth rate is maximal at intermediate density.





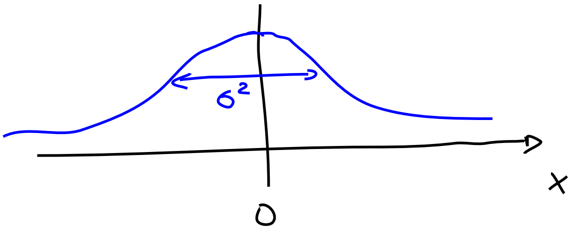


bistability



Dispersal kernels

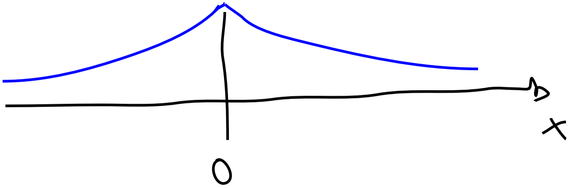
- Gaussian $K(x, y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-y)^2}{2\sigma^2}}$
- Laplace
- movement model derivation



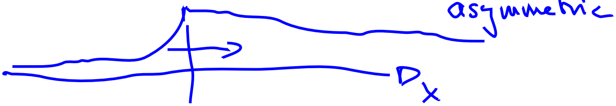
Dispersal kernels

- Gaussian
- Laplace $K(x, y) = \frac{1}{\sqrt{2\sigma^2}} e^{-\sqrt{\frac{2}{\sigma^2}}|x-y|}$
- movement model derivation

$K(x,y) = \tilde{K}(x-y)$



example :



A simple example

Uniform dispersal on an island. $K(x, y) = \frac{1}{|\Omega|}$ gives $\boxed{N_{t+1} = \bar{F}(N_t)}$

$$N_{t+1}(x) = \int_\Omega k(x,y)\, F(N_t(y))\, dy$$

$$= \frac{1}{|\Omega|} \int F(N_t(y))\, dy \qquad |\Omega| = \int_\Omega dx$$

$N_0 = N_0(x) \Rightarrow N_1(x) \equiv N_1$

Uniform dispersal with loss

$N_{t+1} = s\bar{F}(N_t)$ and $F(N) = \frac{RN}{1+(R-1)N}$

$0 \leq s \leq 1$

fraction of indiv. that stay in Ω.

Steady state: $N^* = 0$

$N^* = \dfrac{sR-1}{R-1} > 0$ only if $s > \dfrac{1}{R}$

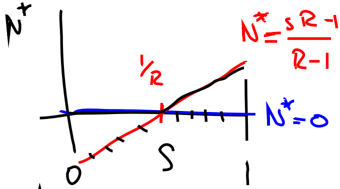
$N^*=0 \to sF'(0) = sR$

0 is stable if $sR < 1$

un- if $sR > 1$

if $s < \frac{1}{R} \Rightarrow N_t \to 0$ as $t \to \infty$

$s > \frac{1}{R} \Rightarrow N_t \to N^* = \dfrac{sR-1}{R-1}$ as $t \to \infty$



Uniform dispersal with loss